Viscosity (VIS)

Topic: Mechanics

1 Key words

Laminar and turbulent flow, Reynolds number, Hagen-Poiseuille's law, Stokes' law

2 Literatur

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D. Meschede, *Gerthsen Physik*, Springer
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3 Basics

3.1 Definition of viscosity

Gedankenexperiment: Assume a liquid between two parallel plates of equal dimensions, in the distance x (cf. fig. 1).. Assume that the space between the two plates is divided into layers of liquid and plate 1 is moved with a constant velocity v. We further assume that the liquid sticks to the plates, i.e. the layer of liquid next to plate 2 remains at rest while the layer next to plate 1 has velocity v. The intermediate layers then have to move with different velocities, such that the velocity increases from the plate at rest to the moving plate. In the simplest case, this velocity distribution is linear.

The lowest layer of liquid exerts a tangential force on the layer directly above it, which thus moves with a velocity v_1. This layer in turn acts on the layer above it, which then moves with velocity v_2. Thus each layer accelerates the next one and at the same time is decelerated by it, following the principle of reaction.

Experiments show that the force F which is necessary to move the plate is proportional to its area A and the velocity v, and inversely proportional to the distance x of the plates:

$$F = \eta \cdot \frac{A \cdot v}{x} \tag{1}$$

The constant η is referred to as *dynamical viscosity* or simple as viscosity. The viscosity can be attributed to the inner friction of the fluid.

The unit used for viscosity η is the Pascal-second (Pa s)

$$1\,\mathrm{Pa\,s} = 1\,\frac{\mathrm{N \cdot s}}{\mathrm{m^2}} = 1\,\frac{\mathrm{kg}}{\mathrm{m \cdot s}} \tag{2}$$

In tables also the old unit Poise (P) can be found.

$$1\ \mathrm{P} = 0,1\ \mathrm{Pa\,s} \quad \text{or.} \quad 1\ \mathrm{cP} = 1\ \mathrm{mPa\,s} \tag{3}$$

The viscosity of gases can be derived and defined in an analog way to the viscosity of liquids.

The viscosity for some fluids and gases are listed in the table 1.

Note that the viscosity of a liquid normally decreases with increasing temperature, while the viscosity of a gas increases. *Therefore giving viscosities without giving the temperature is useless and must be avoided in your report.*

The ratio of the dynamical viscosity η and the density ρ is called *kinematical viscosity*

$$\nu := \eta/\rho \tag{4}$$

(Greek letter nu). ν is given in units of m^2/s, the obsolete unit is Stokes ($1\ \mathrm{St} = 10^{-4}\ \mathrm{m}^2/\mathrm{s}$).

If η is independent of the velocity v, the liquid is referred to as a *Newtonian* fluid, and it has a linear velocity profile mentioned above. Most pure liquids are Newtonian, such as water. In the case that η is a non-constant function of v, the liquid is called non-Newtonian.

While the physical Theory of Newtonian fluids is easier to handle, non Newtonian fluids are more common. Margarine, for example, belongs to the groups of thixotropic fluids. This means its viscosity decreases with increasing shear force. Other thixotropic fluids are ketchup, shaving foam and some paints.

Non Newtonian and non thixotropic are for example honey, condensed milk, printing ink, ink in ball pens, blood and motor oil.

3.2 Laminar and turbulent flow

The case in which the fluid layers float without mutual interferences, i.e. without creating turbulences, is referred to as *laminar flow*.

Figure 2a shows examples of laminar flows between two horizontal arranged glass plates with different obstacles.

The streamlines break on edges of non-streamlined bodies or if the stream velocity is too high. In this case turbulences are generated (cf. fig. 2b). The flow resistance of *turbulent flow* is much larger than that of laminar flow.

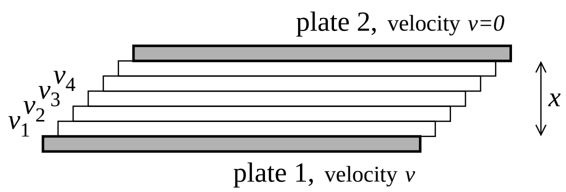
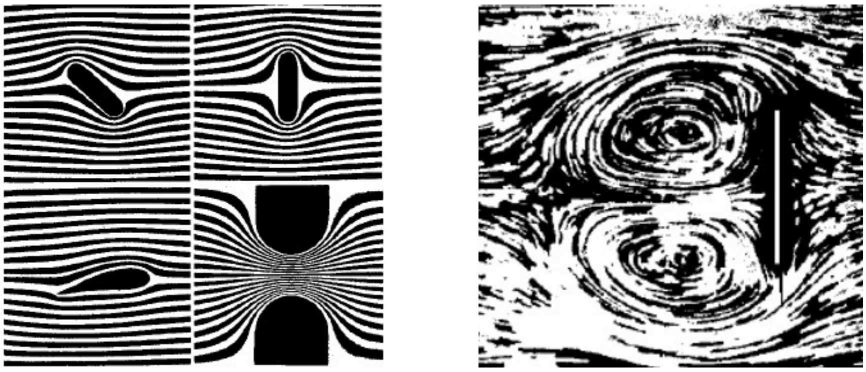


Figure 1:Layers of fluid



(a) (b)

Figure 2: laminar (a) and turbulent flow (b)

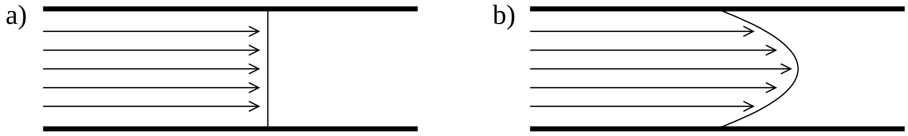


Figure 3: velocity profile in a pipe

3.3 Hagen-Poiseuille's law

The German engineer Hagen (1839) and the French doctor Poiseuille (1840 in an analysis on the blood circuit) discovered independently, that the current i (volume flowing out per time unit) is inversely proportional to the viscosity η and to the length of the pipes l, and directly proportional to the pressure difference $\Delta p = p_2 - p_1$ and to the 4th power of the pipe radius r. The *Hagen-Poiseuille's law* is:

$$i = \frac{V}{t} = \frac{\pi \cdot r^4 \cdot \Delta p}{8 \cdot \eta \cdot l} \tag{5}$$

You can imagine the layers of fluid inside a pipe as a series of pipelines, one inside each other, where the innermost has the highest velocity. When colored water flows through a pipe after clear water with an initially plane boundary layer, the initial current is as shown in figure 6 3a. After some time, the current is as shown in figure 3b. The resulting velocity profile has parabolic shape. The maximum particle velocity can be found in the center of

Table 1: Viscosity of fluids and gases at 20°C and standard pressure

matter	**viscosity (mPa s)**
diethyl ether	0,240
water	1,002
motor oil	100-600
glycerol (pure)	1480 (mixed with water: 10x and more smaller)
air	0,0182
hydrogen	0,0088

the pipe. The flux i is equal to the mean velocity of the fluid v_{mean} times the cross-sectional area A.

$$i = A \cdot v_{\text{mean}} \tag{6}$$

Hagen-Poiseuille's law is only valid under the following idealized conditions (otherwise it only serves as approximation):

1. There is only friction force, no inertia. That means that the particles of the fluid are not accelerated while they are flowing in the pipe. The total work is due to friction. Such a flow is also called steady flow, meaning the flux does not change in time.

2. $v = 0$ at the pipe walls. Only the inner friction of the fluid is taken into account.

3. $\eta =$ const. we are dealing with a Newtonian fluid. The flux-pressure difference diagram shows a straight line.

4. $\rho =$ const, i.e. incompressibility. for fluids this predominanty is valid, for gases it is not.

5. The radius of the pipeis the same within the relevant area, so $r =$ const.

6. There is laminar flow. Inflow turbulences that enhance the flow resistance are not taken into account.

   At too high velocities of the fluid, the laminar flow can break off. As a criterion the *Reynolds number* can be used, which is defined as

   $$Re = \frac{L \cdot \rho_{\text{Fl"ussigkeit}} \cdot v_{\text{mittel}}}{\eta} \tag{7}$$

   with L the characteristic dimension. For a tube flow $L = d = 2 \cdot r$ is the inner diameter of the tube.

   As long as $Re < 2300$ the flow is mostly laminar. For $Re < 1160$ the flow is laminar fore sure, between 1160 and 2300 inflow disturbances may result in turbulent flow, which will enhance the flow resistance.

7. There are no external forces (i.e. horizontal pipes or closed loops).

Since in the Hagen-Poiseulle's formula (5) there are all quantities measurable except η, it is possible to determine the viscosity experimentally.

3.4 Flow resistance

The Hagen-Poiseulle's law (5) can be formulated as

$$\Delta p = W \cdot i \qquad \text{with} \qquad W = \frac{8 \cdot \eta l}{\pi \cdot r^4} \tag{8}$$

W is called *flow resistance*. It is not only dependent on the pipe geometry but also on the viscosity of the fluid. For Newtonian liquids W is the slope in an flux-pressure difference diagram $\Delta p(i)$.

This formula is the analogon to the Ohm's law for electricity. The correspondences are pressure difference Δp and voltage V, flux i and current I, flow resistance W and electric resistance R, and flow and electric conductivity $L = 1/W$ and $L = 1/R$. In a parallel configuration the conductivities are added, in a serial configuration the resistances.

3.5 Stokes' Law

So far ee have discussed the flow of a liquid in a pipe. Now we want to examine a body moving in a fluid. Even here the viscosity plays an prominent role.

We are locking at a sphere (raduis r, velocity v) falling through a infinite volume containing a viscous medium. The following forces act on a sphere (cf. Abb. 4):

1. Gravitation: $F_\mathrm{G} = m \cdot g$
2. Buoyancy: $F_\mathrm{A} = V_\mathrm{sphere} \cdot \rho_\mathrm{liquid} \cdot g$
3. Friction: F_R
   It is always directed against the movement. In the case of laminar flow the friction is given by *Stokes' formula*
   $$F_\mathrm{R} = 6 \cdot \pi \cdot \eta \cdot r \cdot v \tag{9}$$
   In contrast to the flow in a pipe this is only valid for a Reynolds number **Re** $<$ **1**.

   The change from laminar to turbulent flow strongly depends on the shape of the moving object.

If the equilibrium of forces is reached

$$|F_\mathrm{G}| = |F_\mathrm{A}| + |F_\mathrm{R}| \tag{10}$$

we derive the dynamical viscosity η with (9)

$$\eta = \frac{2 \cdot r^2 \cdot g}{9 \cdot v} \left(\rho_\mathrm{Kugel} - \rho_\mathrm{Fl"ussigkeit}\right) \tag{11}$$

If the sphere is moving in an cylinder with radius R instead of an infinite volume, a corrected formula must be used

$$\eta = \frac{2 \cdot r^2 \cdot g}{9 \cdot v \cdot \left(1 + 2,4\,\frac{r}{R}\right)} \left(\rho_\mathrm{Kugel} - \rho_\mathrm{Fl"ussigkeit}\right) \tag{12}$$

For a cylinder of finite length there are additional smaller corrections which are neglected here.

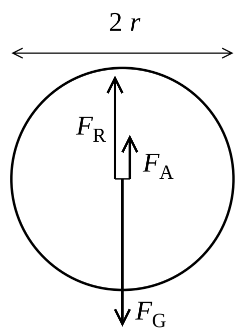


Figure 4:
Forces acting on a falling sphere in a viscose medium.

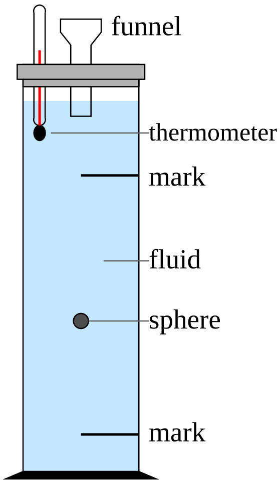


Figure 5:
falling sphere viscosimeter

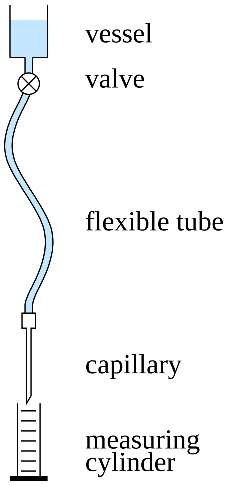


Figure 6:
capillary viscosimeter

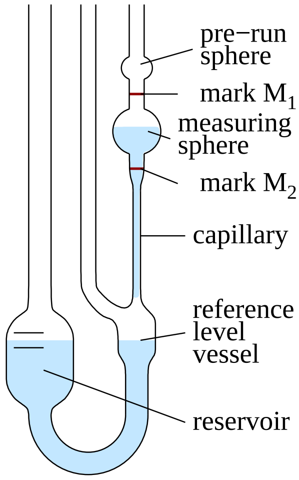


Figure 7:
Ubbelohde viscosimeter

4 Experimental setup

4.1 Falling sphere viscosimeter

A long transparent cylinder with two marks is filled with the liquid to study. Due to a funnel, the trajectory of the spheres is parallel to the cylinder axis (cf. fig. 5).

In this experiment glass spheres are used, their diameter is measured using a digital caliper, their mass using a precision balance.

Other experimental accessories are: areometer to determine the density of the fluid, thermometer, chronometer and rule.

4.2 Capillary viscosimeter

You will use cannulae (for medical injections), available in different types, as thin capillaries. The liquid flows from a vessel that can be changed in height through a flexible tube and a valve, until it reaches the capillary (cf. fig. 6).

You can measure the pressure from the difference between the level of liquid in the store vessel and the needle. The flux through the needle can be easily calculated by measuring volume of the liquid pouring out in a certain time. If in addition the length and the diameter of the needle are measured in addition, one obtains all necessary data to calculate the viscosity of the liquid.

4.3 Ubbelohde viscosimeter

The principle of this precisely calibrated viscometer is the same as a capillary viscometer (cf. fig. 7). From the measuring sphere the fluid runs through the capillary to the reference level vessel. The time the liquid level needs to drop from mark M_1 to M_2 is measured. These marks define both the volume of the sample and the mean hydrostatic pressure.

The viscosimeter is calibrated so that one obtains the kinematic viscosity by multiplying the measured time with the instrument constant K

$$\nu = K \cdot t \tag{13}$$

Knowing the density of the fluid the dynamic viscosity η can be calculated

5 Experimental procedure and analysis

5.1 Falling sphere viscosimeter

Determine the needed variables, in order to calculate the viscosity of the liquid in the vertical tube according to formula (11) and (12).

Pick out at least ten spheres of the same size and measure their diameter and mass. It can be better to weight all spheres together and then divide the mass by the number of spheres.

Drop the spheres through the funnel and measure the time they need to cover the distance between the two marks.

Remove the spheres from the bottom of the tube using the catching sieve and clean them afterwards with distilled water.

Measure also the density of the liquid using the areometer.

Think about the systematic and statistical uncertainties for all measures values.

Analysis

Calculate the viscosity of the glycerol-water-mixture at room temperature according to formula (11) and (12) and give the errors.

Compare your result to values given in the literature (e.g. fig. 8)

Calculate also the Reynolds number and decide whether the flow is laminar.

5.2 Capillary viscosimeter

Your tutor will give you two needles. Note the information on the wrapping.

5.2.1 preliminary experiment

Pull the piston of a 50 ml-syringe up to 50 ml, lock the syringe at the end with your finger as close as possible and press the piston with your thumb as much as possible. Read off the reached volume. If the piston is difficult to operate in the syringe, put some silicone oil on the rubber lips.

Then measure the time you need to push 5 ml of water and 50 ml of air as fast as possible through the two needles. Estimate the lengths of the needles (the length indicated on the wrapping refers to the visible part of the needle). The inside diameter of the needle is approximately half the outside diameter indicated on the wrapping.

Analysis immediately in the lab

Calculate the pressure Δp you reached using the Boyle-Mariotte law and determine the viscosity of water and air with the Hagen-Poiseuille's law. Compare your value with the literature value. Discuss uncertainties (qualitatively).

5.2.2 Flow resistance of two needles

Mount the needle to the lower end of the flexible tube. Check if there is enough deionized water in the vessel.

Measure the difference in height between the needle and the water level in the vessel.

Measure the amount of water flowing through the needle in a certain time. Since the volume of the water in the measuring cylinder can not be read very precisely, it is better to weigh the mass of the cylinder without and with the water using the precision balance. With this and the density of water one can calculate the volume.

Repeat this measurement for *at least four more* significantly different heights of the vessel. Use the full height of the stand.

Repeat these measurements for the second needle.

Heat up the syringe needle with a lighter where it is stuck in a plastic connector and pull it out of the plastic connector with pliers. Measure the length l of the needle with the caliper (from where to where is it reasonable?) and the inside diameter d with the microscope. To find the image of the needle in the microscope can be difficult, if done in a non-systematic way.

Determine the uncertainties for l and d.

Note: deposit the needle into a special trash. Do not throw it into the normal trash.

Analysis

Plot the pressure differences vs. the flux i. Fit a straight line to the data points, determine the slope and its uncertainty. The slope represents the flow resistance W.

The evaluation by using the slope has two advantages: firstly you obtain an average value, secondly errors are eliminated, which add to the pressure like constants (e.g. caused by surface tension or a wrong reading position).

Calculate the viscosity using equation (8) and make an error analysis. Compare your values with the literature values.

5.3 Ubbelohde viscosimeter

This experiment should be done with all groups together.

Switch on the pump of the thermostat and adjust the temperature to the same value as in the previous experiment. Wait about 15 min for equilibration if the temperature was changed.

The probe is filled into the pipe above the reservoir, until it is between the two marks. Make sure that the capillary in the viscometer is in an accurate vertical position.

Lock the pressure compensation tube (middle tube in fig. 7) with the finger and suck the liquid with the water-jet pump into the pre-run sphere. Switch the pump off, separate the tube between the pump and the viscometer and open the pressure compensation tube thereafter.

Measure the flow time between the marks M_1 and M_2, every student by himself. Calculate the mean and the statistical uncertainty from all values. Notice the temperature in the bath as well.

Repeat the experiment for different temperature (ask your tutor). At the end of the experiment, switch off the thermostat.

Read the instrument constant K written on the viscosimeter (unit cSt/s).

Analysis (directly in the lab)

Calculate the kinematic viscosity ν with equation (13). and from this the dynamic viscosity η (eq. (4)).

Compare the values with the result in section 5.2 and with the literature value for the used temperatures (cf. fig. 9).

6 Questions

1. Write the equilibrium of forces for a carbon dioxide bubble in a glass of sparkling water.

2. Calculate the viscosity of air from the maximum velocity a parachuter reaches in free fall (approx. 200 km/h). Make additional assumptions. What do you notice? Calculate the Reynolds number!

3. Calculate the Reynolds number for normal respiration by nose by estimating radius of the nostrils. Normal breathing consists of approximately 15 breathes per minute, each breath approximately 0.5 liters of air (density of air $\rho_{\text{Luft}} = 1,29 \cdot 10^{-3}\text{g/cm}^3$). Is it possible to achieve turbulent flow in the nostrils when intensifying your respiration?

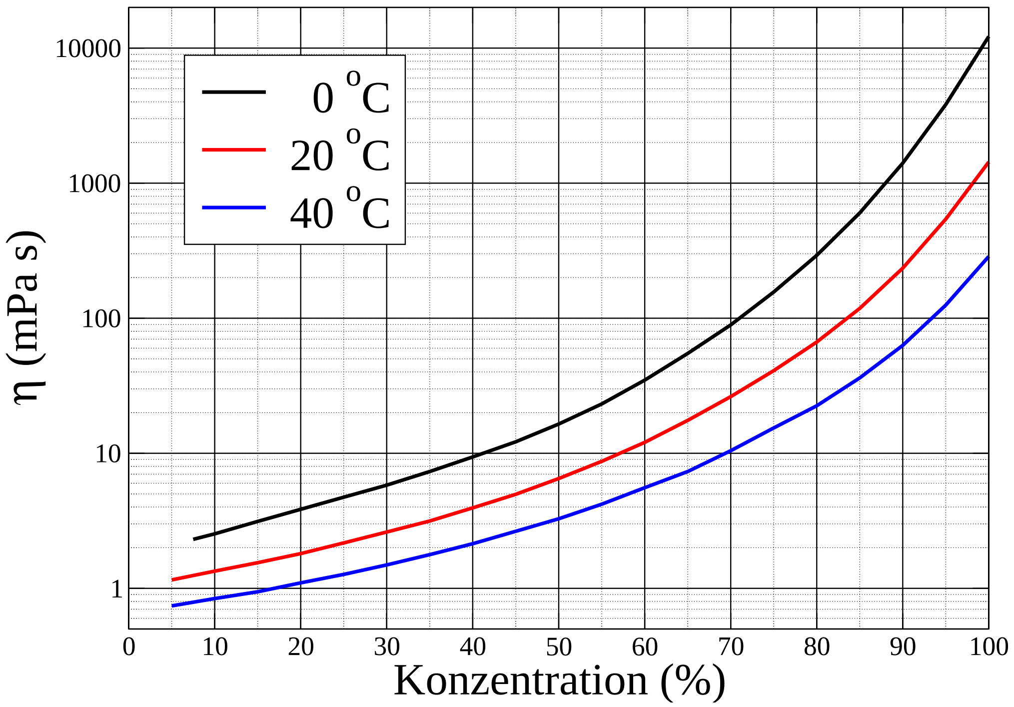


Figure 8: viscosity of a glycerol-water mixture vs. the glycerol concentration

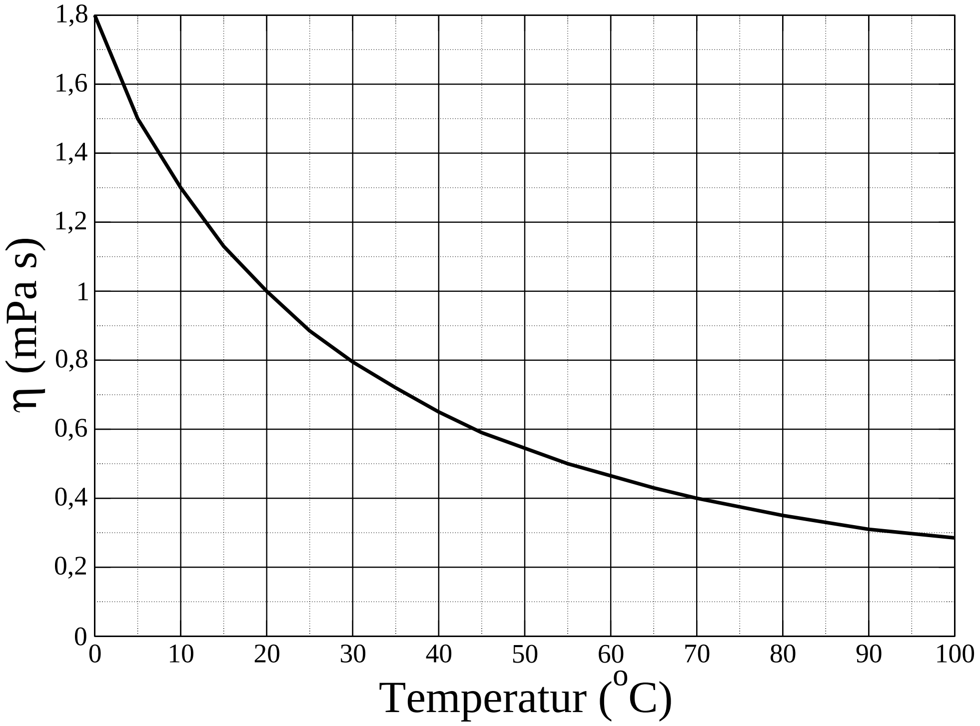


Figure 9: viscosity of water vs. temperature