

Kräfte: $F_G = G \cdot \frac{mM}{r^2}$, $F_F = k \cdot (x - x_0)$, $F_A = \rho_{fl} g V_k$, $F_R = \mu_k F_N$, $F_C = 2m\vec{v} \times \vec{\omega}$, $F = EA \cdot \frac{\Delta L}{L}$ $\sum F = m \cdot \ddot{x}$

Energien: $E_F = \frac{1}{2} k(x - x_0)^2$, $E_{rot} = \frac{1}{2} I \omega^2$, $E_G = G \cdot \frac{mM}{r} = \int F_G dr$, $W = \int F dr = \int F dr \cos(\alpha)$

Wurf/Fall: $t_f = \sqrt{\frac{2h}{g}}$, $v = \sqrt{2hg}$, $v_{flucht} = \sqrt{2Rg}$, $p = m \cdot v = \sqrt{2kmx^2}$, $\text{max. Höhe } r = \frac{R}{1 - \frac{v^2}{2Rg}}$, $x_s = \frac{v^2}{2g} \sin(2\alpha)$
 $\vec{r}(t) = (v_0 \cos(\alpha)t, v_0 \sin(\alpha)t - \frac{1}{2}gt^2 + y_0)$ schiefe Ebene: $v = \sqrt{\frac{2gn}{1 + \mu^2}}$ Bremsvorgang: $v = v_0 \ln\left(\frac{v_0 + u_p}{u_p}\right) - gT$

Rotationsbewegung: $\Delta s = r \cdot \phi$ $v = r \cdot \dot{\phi} = r \cdot \omega$, $\omega = \frac{1}{r} \vec{r} \times \vec{v}$ $a = r \cdot \ddot{\phi} = r \omega^2 = \frac{v^2}{r}$ $\sum M = I \cdot \ddot{\phi}$ $M_D = D \cdot \phi$	Masse m Impuls $p = m \cdot v$ Kraft $F = \dot{p}$ Energie $E = \frac{1}{2} m v^2$ Leistung $P = F \cdot v = \frac{W}{t}$	$I = \int r^2 dm = \int r^2 \rho dV$ Trägheitsmoment $L = I \omega = r \times p$ Drehimpuls $M = \dot{L} = \dot{\phi} = r \times F$ Drehmoment $E = \frac{1}{2} I \omega^2 = \frac{1}{2} L \omega$ Rot.-energie $P = D \cdot \omega$ Leistung	Tensor $\begin{pmatrix} xx & xy & xz \\ xy & yy & yz \\ xz & yz & zz \end{pmatrix}$
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↳ **Direktionsmoment** Trägheitsmomente: Pkt/Horiz/zy: $\frac{1}{2} m r^2$ Voll/zy: $\frac{1}{2} m r^2$ Kugel: $\frac{2}{5} m r^2$

Stoß: $\vec{p}_s = \frac{1}{n} \sum m_i \vec{v}_i = \frac{1}{n} \int \vec{r} dm = \frac{1}{n} \int \vec{r} \rho dV$
dezentral:
 $p_{gesx} = m_1 v_1 \cos \theta_1 + m_2 v_2 \cos \theta_2$
 $p_{gesy} = m_1 v_1 \sin \theta_1 + m_2 v_2 \sin \theta_2$

SP-System:
 $R_s = \frac{u \cdot r \cdot u}{r}$
 $R_s = M \cdot s = u \cdot v_1 + u \cdot v_2$
 $\vec{r}_2 = \vec{r}_1 - \vec{r}_2$

Pendel: $\ddot{\phi} = -\frac{g}{l} \phi \rightarrow T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{I}{M}} = 2\pi \sqrt{\frac{M l^2}{M l}} = 2\pi \sqrt{\frac{l}{g}}$
 $\omega \times = -\omega g \sin \phi = m l \ddot{\phi}$

Bewegungsglg:
 $m \ddot{x} = -kx - b\dot{x} \rightarrow \ddot{x} + 2\gamma \dot{x} + \omega_0^2 x = 0$; $\omega^2 = \frac{k}{m}$
 Kraft Reibung
Torsion: $M = -D\phi = I\ddot{\phi} \Rightarrow \ddot{\phi} + \frac{D}{I}\phi = 0$, $\omega^2 = \frac{D}{I}$

Schwingungen: $x(t) = A \cos(\omega t + \phi) = \frac{1}{2} |c| (e^{i(\omega t + \phi)} + e^{-i(\omega t + \phi)}) = A e^{-\gamma t} \cos(\omega t)$
 $E_{pot} = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$
 $E_{kin} = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t + \phi)$

erzw. Schwingungen:
 1) $\omega \ll \omega_0$, $\omega \ll \frac{b}{m}$ Rückstellermw
 $x(t) = \frac{F_0}{k} \cos(\omega t)$, $\delta = 0$
 2) $\omega \gg \omega_0$, $\omega \gg \frac{b}{m}$
 $-x(t) = \frac{F_0}{m \omega^2} \cos(\omega t)$, $\delta = -\pi$
 3) $\omega = \omega_0$
 $-b \omega_0 A \sin(\omega_0 t + \delta) = F_0 \cos(\omega_0 t)$, $\delta = \frac{\pi}{2}$
 $A = \frac{F_0}{b \omega_0}$

gekoppelt:
 $m_1 \ddot{x}_1 = -k x_1 - q(x_1 - x_2)$
 $m_2 \ddot{x}_2 = -k x_2 + q(x_1 - x_2)$
Überlagerung:
 $F = f_1 + f_2$, $A = 2a \cos(\frac{\omega_1 + \omega_2}{2} t)$
 $x(t) = A \cos(\frac{\omega_1 + \omega_2}{2} t)$

Elastizität
 $\sigma = \frac{F}{A} = E \cdot \frac{\Delta L}{L}$ Zugspannung
 $\frac{\Delta V}{V} = 2 \frac{\Delta w}{w} + \frac{\Delta L}{L} = E(1 - 2\nu)$ Querkontraktion
 $\nu = -\frac{\Delta w/w}{\Delta L/L}$
 $\tau = \frac{F}{A} = G \cdot \alpha$ Scherspannung

Hydr. Presse: $\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow A_1 \cdot s_1 = A_2 \cdot s_2$
Hydr. stat. Druck: $p = \rho_{fl} \cdot g \cdot h$, $p = \frac{F}{A}$
Kontinuitätsglg: $A_1 v_1 = A_2 v_2$

$v_{verdr} = \frac{\partial k}{\partial t} \cdot v_k$
 $\Delta E = G \cdot \Delta A = G \cdot 2l \Delta x$ $\dot{V} = A \cdot v$
Bernoulliglg: $p_0 = p + \frac{1}{2} \rho v^2 + \rho g h = \text{const}$

$v(r) = \frac{p_0 - p_z}{4 \eta l} (R^2 - r^2)$
 $p(h) = p_0 \exp\left(-\frac{\rho g h}{p_0}\right)$
Schallpegel: $L = 10 \log \frac{I}{I_0}$, $I_0 = 10^{-12} \frac{W}{m^2}$

Wellen: $v = \lambda \cdot f = \frac{\omega}{k}$, $k = \frac{2\pi}{\lambda}$ Wellenlg: $\frac{\partial^2 D}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 D}{\partial x^2}$ $D(x,t) = A \cdot \sin(\omega t - kx)$

einseitig: $l = (2n+1) \frac{\lambda}{4}$, $f_k = \frac{1}{4} (2k+1) \frac{c}{l}$	L in Festk: $v = \sqrt{\frac{E}{\rho}}$	T Saite/Kabel: $v = \sqrt{\frac{F}{\mu}}$, $\mu = \frac{m}{L}$
beidse: $l = (n+1) \frac{\lambda}{2}$, $f_k = \frac{1}{2} (k+1) \frac{c}{l}$	L in Fluid: $v = \sqrt{\frac{\gamma}{\rho}}$	Schall: $\sqrt{\frac{\gamma}{\rho}}$
Knoten: $x = \frac{\lambda}{4\pi} ((2n+1)\pi + \phi)$	T in Festk: $v = \sqrt{\frac{E}{\rho}}$	
Bauch: $x = \frac{\lambda}{4\pi} (2n\pi + \phi)$	T Saite: $v = \sqrt{\frac{F}{\rho}}$	

Maxwell'sches Rad:
 $m \ddot{x} = F_G - F_F = m g - I \cdot \frac{a}{r}$
 $\rightarrow a = \frac{g}{1 + \frac{I}{m r^2}}$

Galileo:
 $s' = s - v \cdot t + s_0$
 $s = \frac{1}{2} a t^2 = \omega r t^2 = \omega D t$
 $t = \frac{s}{\omega D} \rightarrow v = \frac{\omega D^2}{s}$

Kepler:
 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 $e = \sqrt{a^2 - b^2}$, $e = \frac{c}{a}$
 $\frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3}$

$v_{end} = \frac{1}{\omega_1 + \omega_2} [(\omega_1 - \omega_2) v_{1anf} + 2 \omega_2 v_{2anf}]$

Math. Erg:

Zylinder:

$$\begin{pmatrix} r \cos \varphi \\ r \sin \varphi \\ z \end{pmatrix}$$

$$r \, dr \, d\varphi \, dz$$

Kugel:

$$\begin{pmatrix} r \cos \varphi \sin \theta \\ r \sin \varphi \sin \theta \\ r \cos \theta \end{pmatrix}$$

$$r^2 \sin \theta \, dr \, d\varphi \, d\theta$$

Feder:

$$\begin{pmatrix} -2k & k \\ k & -2k \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\cos \varphi = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \cdot \|\vec{w}\|}$$

$$\sin \varphi = \frac{|\vec{v} \times \vec{w}|}{\|\vec{v}\| \cdot \|\vec{w}\|}$$

Part. Integration:

$$\int_a^b f'(x)g(x)dx = f(x)g(x) \Big|_a^b - \int_a^b f(x)g'(x)dx$$

Partialbruchzerlegung:

$$\frac{f(x)}{g(x)} = \sum_i \left(\frac{A_i \cdot \omega_i}{(x-a_i)} + \dots + \frac{A_i \cdot (x)}{(x-a_i)^k} \right)$$

Trennung d. Variablen:

$$\frac{dy}{dt} = f(t)g(y) \Rightarrow \int \frac{1}{g(y)} dy = \int f(t) dt$$

Variation d. Konstanten:

$$\dot{x} = a(t)x + b(t)$$

$$\Rightarrow x_p(t) = e^{A(t)} \int b(t) e^{-A(t)} dt$$

$$x_h(t) = C e^{A(t) t_0}$$

$$R_{sp} = \frac{4}{3} \int_V z d^3x$$

$$M = \rho \int_V d^3x$$

$$I = \rho \int_V r^2 d^3x$$

Korrekturfaktoren nicht vergessen

höhere Ordnung:

$$\ddot{x} = -kx \quad \dot{u} = -kx$$

$$\Rightarrow u := \dot{x} \Rightarrow \ddot{u} = A\ddot{u} \text{ mit } \vec{u} = \begin{pmatrix} \dot{x} \end{pmatrix}$$

$$V_{\text{Kegel}} = \frac{1}{3} \pi r^2 h$$

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n$$

$$V_{\text{Kugel}} = \frac{4}{3} \pi r^3$$

$$\text{Skalarprodukt: } \langle \vec{v} | \vec{w} \rangle = \sum_{i=1}^n v_i w_i, \quad \|\vec{v}\| = \sqrt{\langle \vec{v} | \vec{v} \rangle}$$

$$\langle f | g \rangle = \int_a^b f(x)g(x)dx$$

$$\text{Spatprodukt: } \vec{u} \circ (\vec{v} \times \vec{w}) = -\vec{u} \circ (\vec{w} \times \vec{v}) = \vec{w} \circ (\vec{u} \times \vec{v}) = -\vec{v} \circ (\vec{w} \times \vec{u})$$

$$\text{Kreuzprodukt: } \vec{v} \times \vec{w} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$