Allgemeines:

$\sin(x \pm y) = \sin(x)\cos(y) \pm \sin(y)\cos(x)$

$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-\cos(x)}{2}}$

$\cos\left(\frac{x}{2}\right) = \sqrt{\frac{1+\cos(x)}{2}}$

$\sin^2(x) + \cos^2(x) = 1$

$\sin(x)\cos(y) = \frac{\sin(x-y) + \sin(x+y)}{2}$

$\sin(x)\sin(y) = \frac{\cos(x-y) - \cos(x+y)}{2}$

$\cos(x)\cos(y) = \frac{\cos(x-y) + \cos(x+y)}{2}$

$\sin(x) \pm \sin(y) = \begin{cases} 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right), & + \\ 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right), & - \end{cases}$

Kugel: $V = \frac{4}{3}\pi r^3$, $O = 4\pi r^2$

Zylinder: $V = \pi r^2 \cdot h$, $O = 2\pi r^2 + 2\pi r h$

Kegel: $V = \frac{1}{3}\pi r^2 h$, $O = \pi r^2 + \pi r s$, $s = \sqrt{h^2 + r^2}$

$\Delta = \nabla \cdot \nabla$ Beispiel: $\Delta \vec{E} = \frac{\partial^2}{\partial x^2}E_x + \frac{\partial^2}{\partial y^2}E_y + \frac{\partial^2}{\partial z^2}E_z$

Relativitätstheorie:

Galilei-Trafos

$t = t'$, $\vec{r} = \vec{r}' + \vec{u}t$

$\vec{v} = \vec{v}' + \vec{u}$, $\vec{a} = \vec{a}'$

$\vec{F} = \vec{F}'$

Postulate:

- Inertialsys. gleichber.
- c in allen IS gleich

zeitartig getrennt

$\to$ gl. Reihenfolge

$c^2(t_1-t_2)^2 - (x_1-x_2)^2 > 0$

raumartig getrennt:

$c^2(t_1-t_2)^2 - (x_1-x_2)^2 < 0$

Lorentzfaktor:

$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$m(v) = \gamma m_0$

$\vec{p}(v) = \gamma m_0 \vec{v}$

Lorentz-Trafos

$x' = \gamma(x - vt)$, $y' = y$, $z' = z$

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$

$x = \gamma(x' + vt)$

$t = \gamma\left(t' + \frac{v}{c^2}x\right)$

Längenkontraktion:

$\Delta L' = L_1' - L_2' = x_1' - x_2'$

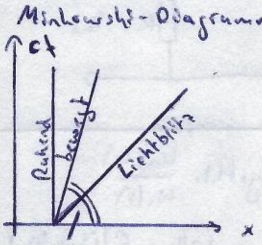
$t_1 = t_2 \to \Delta L' = \frac{1}{\gamma}\Delta L$

Zeitdilatation:

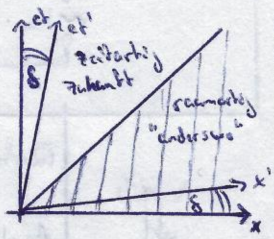
$\Delta t' = t_1' - t_2'$, $x_1 = x_2$

$\to \Delta t' = \gamma \Delta t$

Minkowski-Diagramme:



$\tan\alpha = \frac{ct}{vt} = \frac{c}{v}$



$\tan\delta = \frac{v}{c}$ x'-Achse: $t' = 0$

$\to t - \frac{vx}{c^2} = 0$

Elektrostatik:

$\vec{F}_{el} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r} = q\cdot\vec{E}$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} = \frac{\vec{F}}{q}$, $U = \int_{R_0}^{\infty}\vec{E}(\vec{r})\,d\vec{r}$

$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$

$\Phi_{el}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r}-\vec{r}_i|}\hat{r}$

} Superpositionsprinzip

Satz von Gauss:

$\oint_{\partial V}\vec{E}\cdot d\vec{A} = \int \operatorname{div}\vec{E}\,dV = \frac{1}{\varepsilon_0}\int \rho\,dV = \frac{Q_{in}}{\varepsilon_0}$

Beziehungen:

$\vec{E} = -\nabla\Phi_{el}$, $\Phi = -\int_{r_0}^{r}\vec{E}\cdot d\vec{s}$, $U = \Phi_1 - \Phi_2 = \int_{r_1}^{r_2}\vec{E}\cdot d\vec{s}$

$W = q\cdot U = q\int_{P_1}^{P_2}\vec{E}\,d\vec{s} = \int_{P_1}^{P_2}\vec{F}\,d\vec{s}$

Pkt.ladung: $E(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$ $\Phi_{el}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\vec{r}|}\hat{r}$

Dipolmoment:

$\vec{p} = Q\vec{d}$, im hom. Feld: $\vec{D} = \vec{p}\times\vec{E}$, $E_{pot} = \int\vec{D}\,d\vartheta = -\vec{p}\cdot\vec{E}$ $\to$ E-Feld

$\vec{p}_m = I\cdot\vec{A}$, Drehmoment $\vec{D} = \vec{p}_m\times\vec{B}$, $W = -\vec{p}_m\vec{B}$ $\to$ B-Feld

Kondensator:

Oberflächenladungsdichte $\sigma = \frac{Q}{A}$, mit $\int_{\partial V}\vec{E}\,d\vec{A} = EA = \frac{Q}{\varepsilon_0} \to E = \frac{\sigma}{\varepsilon_0}$

$C = \frac{Q}{U} = \varepsilon_0\varepsilon_r\frac{A}{d}$, $E = \frac{F}{q} = \frac{U}{d}$, $W = \frac{1}{2}CU^2 = \frac{Q^2}{2C}$, $w_{el} = \frac{W}{V} = \varepsilon_0\varepsilon_r\frac{E^2}{2}$

Kugelkondensator:

$U = \frac{Q}{4\pi\varepsilon_0}\frac{R_2 - R_1}{R_1 R_2}$ $C = 4\pi\varepsilon_0\frac{R_1 R_2}{R_2 - R_1}$ $\to$ für gel. Kugel $R_2 \to \infty$

$\to C = 4\pi\varepsilon_0 R_1$

Kondensator mit Dielektrikum:

$E_D = \frac{E_V}{\varepsilon_r} = \frac{\sigma}{\varepsilon_0\varepsilon_r} = \frac{Q}{\varepsilon_0\varepsilon_r A}$, $C_D = \varepsilon_r C_V$, $W = \frac{1}{2}CU^2 = \frac{1}{2}\varepsilon_0\varepsilon_r A d E^2$

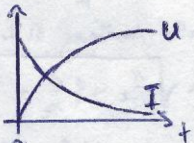
$w_{el} = \frac{1}{2}\varepsilon_0\varepsilon_r E^2 = \frac{1}{2}ED$

Laden und Entladen: (Kondensator)

Aufladen: $U_C = U - RI = U - R\dot{Q}$

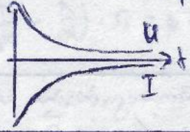
$\to U = R\dot{Q} + \frac{Q}{C}$, $Q_{part} = CU = \text{konst.}$

$\to Q(t) = (Q_0 - U_0 C)e^{-\frac{t}{RC}} + U_0 C$



Entladen: $U_R = -U_C \to \dot{Q} + \frac{Q}{RC} = 0$

$\to Q(t) = Q_0 e^{-\frac{t}{RC}}$



Dielektrika im E-Feld:

$\vec{P} = \frac{1}{V}\sum\vec{p}_i = \varepsilon_0\chi\vec{E}_D = \varepsilon_0(\varepsilon_r - 1)\vec{E}_D$, $\chi = \varepsilon_r - 1 = \frac{\mu_\alpha}{\varepsilon_0}$

$\vec{E}_D = \frac{\vec{E}_V}{\varepsilon_r} = \frac{1}{1+\chi}\vec{E}_V$, χ: elektrische Suszeptibilität, $\vec{P}$: Polarisation

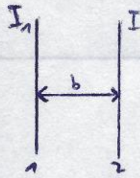
dielektrische Verschiebung: $\vec{D} = \varepsilon_0\vec{E}_D + \vec{P} = \varepsilon_0\vec{E}_D + \varepsilon_0\chi\vec{E}_D = \varepsilon_0\varepsilon_r\vec{E}_D$

$P = \frac{Q_P}{A} = \varepsilon_0(\varepsilon_r - 1)\frac{U}{d}$

Magnetismus:

$\vec{B} = \mu_0\vec{H} = \operatorname{rot}\vec{A}$, $\operatorname{div}\vec{B} = 0$, $\operatorname{rot}\vec{B} = \mu_0\vec{j}$, $\vec{A}$: Vektorpotential, z.B.: $\frac{\mu_0 I}{4\pi}\int_{S_1}\frac{d\vec{s}_1}{r_{12}}$

Lorentz-Kraft: $\vec{F}_L = q(\vec{E} + \vec{v}\times\vec{B})$, auf Draht: $d\vec{F} = I(d\vec{L}\times\vec{B})$, auf Leiter: $\vec{F} = I(\vec{L}\times\vec{B})$



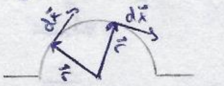
$\frac{F_1}{\Delta L} = I_1\frac{\mu_0 I_2}{2\pi b}$; $\vec{B}$-Feld von ∞-langen geraden Leitern: $\vec{B} = \frac{\mu_0 I}{2\pi R}$

magnetischer Fluss: $\Phi_m = \int\vec{B}\cdot d\vec{A} = L\cdot I$

Hall-Effekt: $U_H = -\frac{|\vec{j}\times\vec{B}|\cdot\vec{b}}{nq} = \frac{IB}{nqd} = R_H\cdot I$

Ampersches Gesetz: $\int_{\partial A}\vec{B}\cdot d\vec{s} = \mu_0 I_{in} = \mu_0\int_A\vec{j}(\vec{r})\,dA = \mu_0 j A$

Biot-Savartsches-Gesetz: $\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{x}\times\vec{r}}{|\vec{r}|^3}$ mit z.B.:



alternativ: $\vec{B}(\vec{r}) = -\frac{\mu_0 I}{4\pi}\int\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\times d\vec{r}'$ $\vec{r}'$ beschreibt die Form der Bahn

Leistung und Strom

$I = \frac{dQ}{dt} = \frac{\sigma_{el}}{L}A\cdot U = nqA\vec{v}_D$

Stromdichte: $\vec{j} = \frac{I}{A} = env_D = \frac{nq^2\tau_s}{m}\vec{E} = \sigma_{el}\vec{E}$

mit $\frac{\Delta Q}{\Delta t} = \frac{en\Delta V}{\Delta t} = \frac{enA\,dt}{\Delta t}v_D = enAv_D = I$

Ohmsches Gesetz: $I = \frac{\sigma A}{L}U = \frac{U}{R}$

elektrischer Widerstand: $R = \frac{1}{\sigma_{el}}\frac{L}{A} = \rho_s\frac{L}{A}$

Leistung: $P = U\cdot I = \frac{U^2}{R} = RI^2$

Arbeit: $W = \int_{t_1}^{t_2}P\,dt$

n: Teilchenanzahl / Volumen

σ_{el}: el. Leitfähigkeit

$\sigma_{el} = nq\mu = \frac{1}{\rho_s}$

$\mu = \frac{v_D}{E}$: Beweglichkeit

$\tau_s = \frac{\Lambda}{v}$: mittl. Stoßzeit

$\bar{v} = \sqrt{\frac{8k_B T}{\pi m}}$ (ohne E-Feld)

Λ: mittl. freie Weglänge

$\rho_s = \frac{1}{\sigma_{el}}$: spez. Widerstand

zugehörige Regeln:

Maschenregel: $\sum_k U_k = U_{Generator}$

Knotenregel: $\sum_k I_k = 0$

Parallelschaltung:

$\frac{1}{R_{ges}} = \sum_i\frac{1}{R_i}$

$C_{ges} = \sum_i C_i$

Reihenschaltung:

$R_{ges} = \sum_i R_i$

$\frac{1}{C_{ges}} = \sum_i\frac{1}{C_i}$

Maxwell-Gleichungen:

Vakuum:

$\operatorname{div}\vec{E} = \frac{\rho_{el}}{\varepsilon_0}$, $\operatorname{div}\vec{B} = 0$

$\operatorname{rot}\vec{E} = -\frac{\partial}{\partial t}\vec{B}$

$\operatorname{rot}\vec{B} = \mu_0\vec{j} + \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E}$

Materie:

$\operatorname{div}\vec{D} = \rho_{el}$, $\operatorname{div}\vec{B} = 0$

$\operatorname{rot}\vec{E} = -\frac{\partial}{\partial t}\vec{B}$

$\operatorname{rot}\vec{H} = \vec{j} + \frac{\partial}{\partial t}\vec{D}$

Integralform:

$\oint\vec{E}\cdot d\vec{s} = -\frac{d}{dt}\Phi_m$

$\oint\vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0\varepsilon_0\frac{d}{dt}\Phi_{el}$

$\oint\vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$\oint\vec{B}\cdot d\vec{A} = 0$

Verschiebungsstrom: $\vec{j}_V = \varepsilon_0\frac{\partial}{\partial t}\vec{E}$, $\frac{1}{c^2} = \mu_0\varepsilon_0$, $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$ ≙ Polarisation

Magnetisierung:

$\vec{M} = \frac{1}{V}\sum\vec{p}_m = \chi_m\vec{H}$ χ_m: Suszeptibilität (magn.), $\mu_r = 1 + \chi_m$

$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(\vec{H} + \chi_m\vec{H}) = \mu_0\vec{H}(1 + \chi_m) = \mu_0\mu_r\vec{H}$

Diamagnetismus: $\chi_m < 0 \to$ Kraft in Richtung des schwächer werdenden B-Felds

Paramagnetismus: $\chi_m > 0$, $|\chi_m| \ll 1 \to$ Kraft in R. d. stärker werdenden B-Felds

Ferromagnetismus: $\chi_m > 0$, $|\chi_m| \gg 1$

Spule:

$L = \mu_0 \mu_r \frac{N^2 A}{l}$ L: Induktivität

$W = \frac{1}{2} L I^2$ A: Fläche

$w_m = \frac{1}{2}\mu_0 H^2 = \frac{B^2}{2\mu_0}$ l: Länge

N: Windungsanzahl

$U_I = -L\frac{dI}{dt} = -\dot{\Phi} N$

$B = \mu_0 \mu_r \frac{I N}{l}$

$w_{em} = \frac{1}{2}(E D + B H)$

Ein-/Ausschalten (Spule):

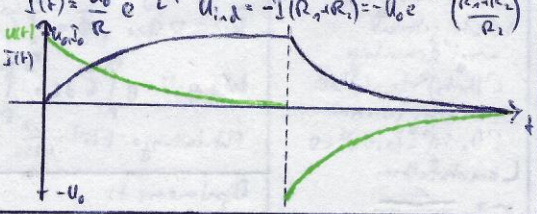
$U_{ind} = -L\dot{I}$

Einschalten: $U_0 = RI - U_{ind} = RI + L\dot{I}$

$I_{part} = \frac{U_0}{R} \Rightarrow I(t) = \frac{U_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

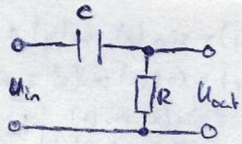
Ausschalten: $U_0 = 0 = RI + L\dot{I}$

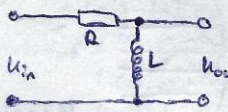
$I(t) = \frac{U_0}{R} e^{-\frac{R}{L}t}$ $U_{ind} = -I(R_1+R_2) = -U_0 e^{-\frac{R}{L}t}\left(\frac{R_1+R_2}{R_1}\right)$



Filter:

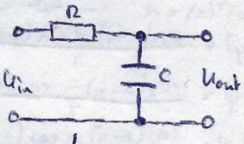
Hochpassfilter:

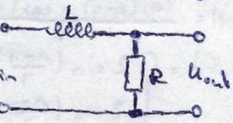




$RI + L\dot{I} = U_{in}$ $U_{out} = L\dot{I}$

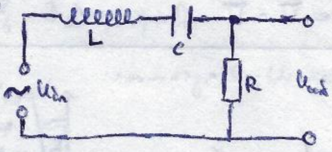
Tiefpassfilter:





$Z \cdot I = U_{in}$ $U_{out} = \frac{R}{Z} U_{in} = R \cdot I$

Bandpassfilter:



zeitabhängige E-Felder:

Faraday'sches Induktionsgesetz: $U_{ind} = -\frac{d}{dt}\Phi_m = -\frac{d}{dt}\int_A \vec{B}\cdot d\vec{A}$, $\text{rot}\,\vec{E} = -\frac{d}{dt}\vec{B}$

Selbstinduktion: $U_{ind} = -L\frac{dI}{dt} = -L\cdot\dot{I}$

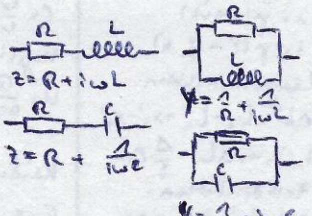
komplexe Widerstände im Wechselstromkreis:

$U = Z\cdot I$ mit $Z = \begin{cases} R & \text{Ohm'scher Widerstand} \\ Z_L = i\omega L & \text{Spule} \\ Z_C = \frac{1}{i\omega C} & \text{Kondensator} \end{cases}$

$\hookrightarrow Z = \frac{U(t)}{I(t)}$

$Y = \frac{1}{Z}$: Admittanz

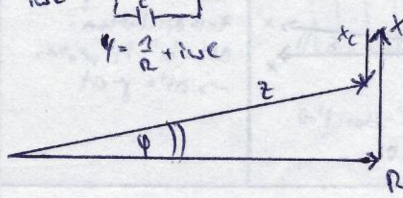
Ansatz: $U(t) = U_0 e^{i\omega t}$



Reihenschaltung R, L, C:

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ $X_C = \frac{1}{\omega C}$, $X_L = \omega L$

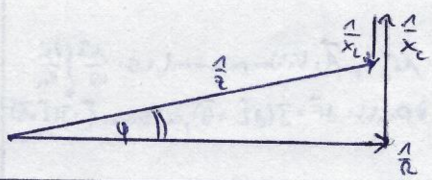
$\tan\varphi = \frac{X_L - X_C}{R}$



Parallelschaltung R, L, C:

$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$

$\tan\varphi = R\cdot\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$



Übertragungsfkt. $\frac{U_{out}(t)}{U_{in}(t)}$

Amplitudenverhältnis $\hat{=}$ |Übertr.fkt.|

Phasenverschiebung:

$\tan\varphi = \frac{\text{Im(Übertrg.fkt.)}}{\text{Re(Übertrg.fkt.)}}$

elektromagnetische Wellen:

$\varepsilon_0\mu_0 = \frac{1}{c^2}$, $\Delta\vec{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}$, $\Delta\vec{B} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{B}$

periodische Wellen: $\vec{E} = \vec{E}_0 \sin(k(z - ct)) = \vec{E}_0\sin\left(kz - \frac{2\pi c}{\lambda}t\right) = \vec{E}_0\sin(kz - \omega t)$, $|\vec{k}| = k = \frac{2\pi}{\lambda}$, $\vec{k} = \frac{\omega}{c}\vec{n}$, $|\vec{n}| = 1$

magn. Feld einer EM-Welle: $\vec{B} = \frac{1}{\omega}(\vec{k}\times\vec{E})$

Intensität: $I = c\varepsilon_0 E^2 = \frac{1}{2}c\varepsilon_0 E_0^2 \longrightarrow$ für $\vec{E}$ periodisch

$w_{em} = \frac{1}{2}\varepsilon_0(E^2 + c^2B^2) = \varepsilon_0 E^2 = \frac{W}{V}$ Energiedichte $W = \int_V \varepsilon_0 E^2\, dV$: Feldenergie im Volumen V; Massendichte $\rho = \frac{w_{em}}{c^2} = \frac{\varepsilon_0}{c^2}E^2$; Impulsstromdichte $\vec{\pi}_{St} = \frac{\varepsilon}{c}\vec{S} = \varepsilon_0(\vec{E}\times\vec{B})$; $p_{St} = c\cdot|\vec{\pi}_{St}| = w_{em}$

Poynting-Vektor: (Richtung und Dichte des Energietransports): $\vec{S} = \vec{E}\times\vec{H} = \varepsilon_0 c^2(\vec{E}\times\vec{B})$ $|\vec{S}| = S = \varepsilon_0 c^2|\vec{E}||\vec{B}| = \varepsilon_0 c E^2 = I$

Vakuum

Dispersionsrelation: $|\vec{k}|^2 = \frac{n^2\omega^2}{c^2}$ $n^2 \sim \varepsilon$ (Brechungsindex)

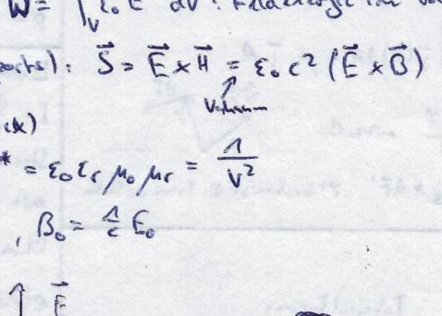
Wellenwiderstand: $Z = \sqrt{\frac{L^*}{C^*}}$ $L^* = \frac{L}{l}$, $C^* = \frac{C}{l}$ $C^* L^* = \varepsilon_0\varepsilon_r\mu_0\mu_r = \frac{1}{v^2}$

E-B-Feld-Zusammenhang: $\vec{B} = \frac{1}{\omega}(\vec{k}\times\vec{E}) \underset{\omega = c|\vec{k}|}{=} \frac{1}{c}\frac{\vec{k}}{|\vec{k}|}\times\vec{E}$, $B_0 = \frac{1}{c}E_0$

Ansatz für ebene Wellen: $E(r,t) = E_0 e^{i(\omega t - kx)}$

$v = \frac{1}{\sqrt{\mu_0\varepsilon_0\mu_r\varepsilon_r}}$

$c = \frac{\omega}{k} = \lambda f = \frac{\lambda\omega}{2\pi}$



$\vec{E}\perp\vec{B}\perp\vec{k}$

Polarisationsarten:

linear: $\left.\begin{aligned} E_x &= E_{0x}\cos(\omega t - kz) \\ E_y &= E_{0y}\cos(\omega t - kz)\end{aligned}\right\}\ \varphi = 0$

zirkular: $\left.\begin{aligned} E_x &= E_0\cos(\omega t - kz) \\ E_y &= E_0\cos\left(\omega t - kz - \frac{\pi}{2}\right) \\ &= E_0\sin(\omega t - kz)\end{aligned}\right\}\ E_{0x} = E_{0y},\ \varphi = \frac{\pi}{2}$

Rechtsschraube: σ^+-Licht

Linksschraube: σ^--Licht

Coulombkraft Elektronenstrahl:

$\frac{1}{2}mv^2 = q\cdot U \longrightarrow v$

$I = \frac{dQ}{dt} = \frac{dQ}{dl}\frac{dl}{dt} = \lambda\cdot v \longrightarrow \vec{E} \rightarrow F_C = q E = \Delta l\cdot\lambda\cdot\vec{E}$