Experiment Number 19
The Transmission of β–Radiation Through Matter

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When electrons are moving through matter, they can lose kinetic energy and leave their original path (Scattering).

The scattering of β–Radiation electrons with an energy range between 1keV and 10MeV is caused mainly by elastic scattering off the nucleus of the target material. The major source of energy loss of the β–Radiation is due to its interaction with the electrons of the target material.

Processes of Interaction without Energy Loss

The elastic scattering of the electrons while they are passing through matter can be classified by the following because of the various measured scattering angles of the electrons:

1. Single Scattering
2. Multiple Scattering
3. Diffusion

If the thickness d of the target is very small, then we have

$$d << \frac{1}{\sigma n} \quad or \quad \rho d << \frac{1}{\sigma}\frac{A}{N_A}$$

where

σ	is the total cross-section of the single scattering process
n	is the number of scattering centres per volume
A	is the mass number of the target atoms
N_A	is the Avogadro's number
ρ	is the density

so that the probability of a certain electron being scattered is very small, which means we have classification 1, Single Scattering. In this case, the Rutherford scattering formula is used to make a quantitative analysis (with certain corrections in the case of relativistic electron speeds).

A larger thickness of the target, $\rho d >> \frac{1}{\sigma}\frac{A}{N_A}$, leads to a larger scattering probability. This is the case of Multiple Scattering with the electrons of the target atoms. A Gaussian distribution can describe the measured distributions of scattering angles very well and is given by

$$W(\theta) \sim \exp\left[-\frac{\theta^2}{2\lambda^2}\right]$$

where

$W(\theta)d\Omega$	is the probability that an electron with a scattering angle θ is scattered in a solid angle $d\Omega$
λ	is the mean statistical uncertainty of the scattering angle

This dependency can be understood if one thinks of the measurement uncertainty of measuring values around the median value. The theory of probability leads to a Gaussian distribution by the assumption of statistical independence and by the assumption of the independence of the different influences (Law of the large number). With this law of the large number, one can understand that the Gaussian distribution is only obtained when a certain minimum thickness of the target is reached. The median scattering value is also proportional to the square root of the number of influences, which means λ should be proportional to $\sqrt{\overline{N}}$ where $\overline{N}$ is the median number of scattering events. Actually, a dependency of the form $\lambda \sim \sqrt{d}$, where d is the thickness of the target is visible. This formula is valid down to median scattering angles of approximately θ ≈20°. This range can be increased if the mean true path of the electrons is used and not the geometric thickness d of the target $\left(\frac{\overline{s}}{d} > 1\right)$.

For an even larger thickness than the normal thickness d_n, the distribution of scattering angles demonstrates no further change and only the transmitted intensity is decreasing. Under these circumstances, the electron transport can be described as a diffusion process. The distribution of scattering angles is approximately of the form $\frac{W(\theta)}{W(0)} \sim \cos^2\theta$. Finally, for electrons with scattering angles greater than 90°, a certain percentage of these electrons are emitted on the incoming side of the target due to multiple scattering. The number of backscattered electrons reaches a value of saturation for a certain thickness d_s (Back-diffusion thickness), where measurements show that $d_s > d_n$.

Inelastic Scattering Interactions

a) The interaction of the incoming electrons with the electrons of the target atom is characterized by a very small mean energy that is transferred at every single scattering process, which is also very small compared to the β-Energy. The total energy loss after passing through the target is also the result of a large number of scattering processes with a small loss of energy at each single scattering process.

   In principle, the inelastic impact of an electron in a solid can be separated in two processes:
   - Energy transmission from the incoming particle on anyone of the shell electrons of the target atom results in an ionization or excitation of the atom or
   - Transmission of an energy quantum collectively on a majority of electrons, which means an excitation of a local quantized oscillation of the electron plasma, which means plasmon

These collective scattering processes contribute a small percentage to the total energy loss for a thick target as compared to the individual processes.

For non-relativistic speeds, the mean energy loss per unit length dx in the target is (according to Bethe and Möller):

$$\frac{\overline{dE}}{dx} = \frac{2\pi e^4 nZ}{E}\left\{ ln\left(\frac{2m_e \mathrm{v}^2}{I}\right) - 1.23 \right\}$$

where n is the number of atoms per volume and v is the electron speed.
Therefore, the specific energy loss is proportional to the electron density of the target.
The mean ionization energy I of the atoms is in first approximation proportional to Z. For relativistic speeds, an additional term is added to the formula above. After passing through a target of thickness d, a mean energy E_m is measured for which

$$E_i^{\,2} - E_m^{\,2} = bd$$

where E_i is the incoming energy and b(Z, E_i) is a material constant.

b) At very high electron energies (MeV range) there are inelastic scattering processes because of the emission of bremsstrahlung during the interaction of the nuclei of the target. The energy loss is given by

$$\frac{\overline{dE}_{st}}{dx} = nZ^2\left(E + mc^2\right)\left\{ 4\ln\frac{2\left(E + mc^2\right)}{mc^2} - \frac{4}{3} \right\}$$

The energy loss due to radiation processes and impact processes are equal at an energy of $E_{kr} = 1.6 \times 10^3 \frac{mc^2}{Z}$, that means the energy loss has a minimum at E_{kr}.

E_{kr} (Al) = 47MeV, E_{kr} (Pb) = 6.9MeV

Range of Electrons in the Target

There are several possibilities to measure the range of mono-energetic electrons: Either one measures the intensity of the transmitted particles in dependence of the layer thickness and extrapolates the linear part of the curve to intensity zero (extrapolated or effective range, Figure 1)

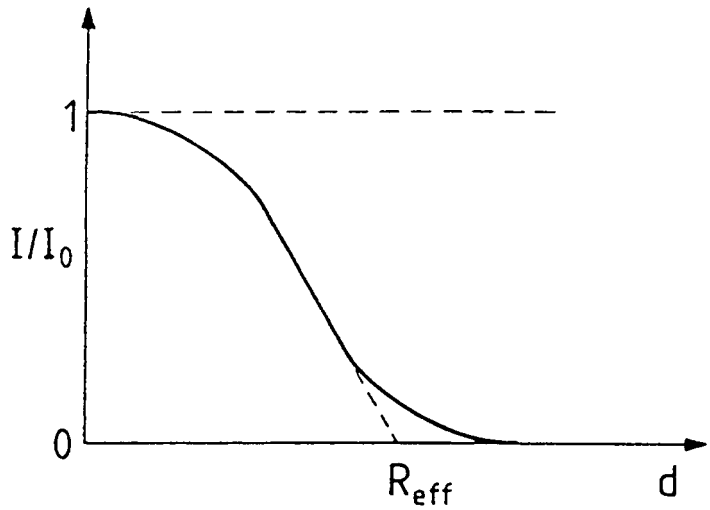

Figure 1: Measurement of the effective range

or one measures the intensity at a given absorber thickness while varying the incoming energy and extrapolating the intensity to zero (Figure 2), where the last method gives the mean range R_m.

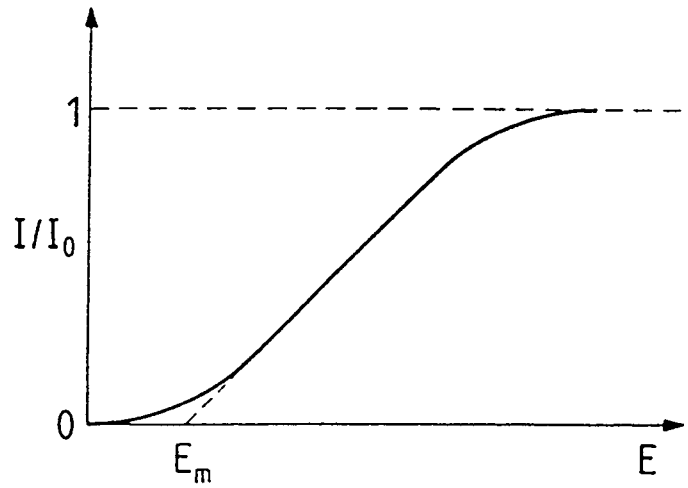

Figure2: Measurement of the mean range

For a small thickness d of the target, the incoming electrons are only scattered at a small angle, but they still lose energy. This is the reason why the scattering cross-section is increasing. Therefore, the intensity is decreasing slowly at the beginning, but decreases linearly with the thickness later on.

With the absorption of β-radiation (continuous energy spectrum), an approximate exponential decrease of the transmitted intensity with increasing thickness of the absorber is visible.

In this case, the target thickness that weakens the electron intensity by a factor of 1000 is defined as the effective range of the electrons. An empirical formula is given by

$$\rho R_{eff}(g/cm^2) = [0.54 \cdot E_{\max}(MeV) - 0.13], \qquad E_{\max} > 0.8\,MeV$$

$$\rho R_{eff}(g/cm^2) = 0.41 \cdot E_{\max}^{1.38}(MeV), \qquad 0.15\,MeV < E_{\max} < 0.8\,MeV$$

E_{max} means the maximum energy of the β-particles

Pre-requisite Knowledge of Experiment

1. Content of the Text
2. Principle of Geiger-Müller and Proportional Counting Tube
3. The Physics of β-decay

Measuring Procedure

Materials: A Geiger-Müller counting tube, a counting apparatus, a ^{90}Sr source, and some target foils are available

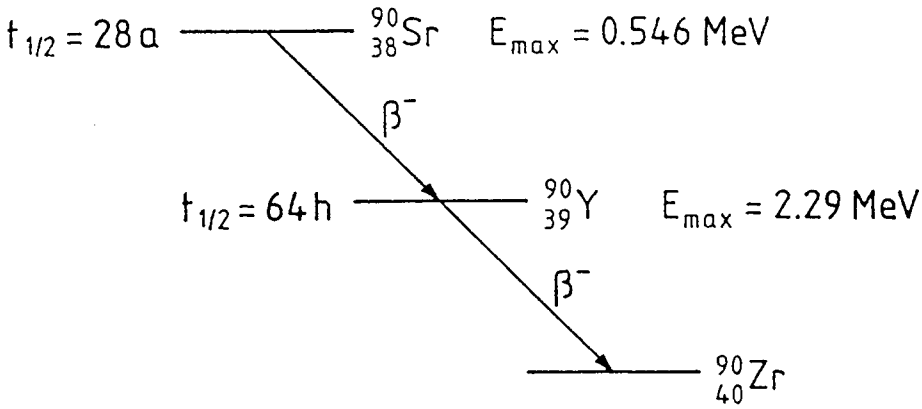

Figure 3: Decay Scheme of ^{90}Sr

1. Measurement of the plateau of the Geiger-Müller counting tube (100 – 1000V)
2. Measurement of the effective range for electrons from the β-source in Aluminum (minimum of 10 measuring points, statistical uncertainty should be less than 3% if possible)
3. Measurement of the back diffusion thickness d_s on Aluminum and Lead
4. Measurement of the back-scattering intensity for three different materials (Al, Ag, Pb) with equal mass allocation
5. What is the relation between the three counting rates to each other?

   (Hint: The counting rate $\dot{dN}$ is given by $\dot{dN} = I \cdot nd \frac{d\sigma}{d\Omega}$, where I($s^{-1}$) is the incoming particle flux, $nd = \tilde{x} \cdot \frac{N_A}{A}$ is the number of target atoms/cm^2 with the mass allocation $\tilde{x} = \rho \cdot d(g/cm^2)$, $\frac{d\sigma}{d\Omega}$ is the differential cross-section and dΩ is the solid-angle covered by the detector)

6. Measurement of the distribution of scattering angles in the range of $0° < \theta < 70°$ of Al targets with thicknesses 0, 0.3, 0.5, and 1mm
7. Background measurement of the exercises 2. and 4.

Report

The report for this experiment is a small but complete scientific paper, which means a natural scientist should be able to understand the paper without any further reading material. The report should contain:

- Short explanation of counting tube's function
- Some sentences about the Physics of β-decay
- Complete protocol of the measurement
- Analysis of the measurements with graphical plots (label and number coordinates with units), measuring points with error bars, results of the measurement and their failures emphasized

Literature

1. K. Siegbahn, α-, β-, γ-Ray Spectroscopy Vol. 1
2. P. Marmier and E. Shelton, Physics of Nuclei and Particles, Vol. 1
3. T. Mayer Kuckuck, Kernphysik
4. H. Neuert, Kernphysikalische Meßverfahren